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Solutions of a sub-Riemannian equation which arises in brain activity modelling

We propose to study properties of solutions to a modification in a sub-Riemannian setting of the classical mean field activity equation proposed by Wilson and Cowan.

Wilson and Cowan first proposed a mean field, equation which models the activity of populations of neurons and their interaction (see [WC]). It was proved that in presence of a visual stimulus and sufficiently strong global inhibition, the solution of the equation is a traveling wave propagating outwards from the stimulus location [A]. In these first models the cortex was modelled as a two-dimensional layer. Later on, the visual cortex was modelled as a fiber bundle, or Lie groups, in order to take into account, the functional geometry of horizontal connectivity ([PT], [CS]). Consequently, also the Wilson and Cowan equation has been reformulated in the Lie group setting, equipped with a sub-Riemannian metric.

This equation in this setting was considered by [SC] to show its ability of reproduce visual perceptual phenomena like figure-ground segmentation and individuation of precepts'. In this project we study properties of the solutions, when the equation and the connectivity kernel are expressed in the geometry of the stereo vision, for its interest in application to 3D visual illusion and completion. Possible approaches are the classical one proposed by [A], or the one proposed in [CCS], for a mean field Kuramoto model.

The ideal candidate should have previous experience in studying brain models in sub-Riemannian setting and/or properties of integro-differential equations.

References

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